

Tethered membranes with long-range self-avoidance: large-dimension limit

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1992 J. Phys. A: Math. Gen. 25 L469

(<http://iopscience.iop.org/0305-4470/25/8/015>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.62

The article was downloaded on 01/06/2010 at 18:22

Please note that [terms and conditions apply](#).

LETTER TO THE EDITOR

**Tethered membranes with long-range self-avoidance:
large-dimension limit**

Pierre Le Doussal†

Institute for Advanced Study, Princeton NJ 08540, USA

Received 17 December 1991

Abstract. The influence of long-range self-avoiding forces on the statistical mechanics of a D -dimensional tethered membrane is studied in the limit of large embedding space dimension $d \rightarrow \infty$. We find three distinct isotropic (crumpled) phases and several flat phases, and thermally driven crumpling transitions for $D > 2$. The effect of attraction and of short-range repulsion for $D > 2$ is also discussed.

There is currently much interest in understanding the effect of steric repulsion on the statistical mechanics of polymerized membranes. These membranes are usually modelled by *tethered surfaces* [1] where the constituent particles are bonded to form a D -dimensional permanent network moving in an external d -dimensional space. One controversial and mathematically difficult question is whether the crumpling transition from a low-temperature flat phase to a high-temperature isotropic phase, which does occur in phantom (e.g. self-intersecting) membranes [1], survives for physical membranes ($D = 2$, $d = 3$) when self-avoidance is taken into account [2]. Extensive numerical simulations [3] show that a large class of self-avoiding tethered surfaces are flat at all temperatures, although with an anomalous roughening exponent of transverse fluctuations. However, recent experiments [4] as well as numerical simulations [5] suggest that a crumpled state might exist. Since the upper critical dimension for self-avoidance in the crumpled phase of phantom membranes $d_u(D) = 2D/(2-D)$ is infinite for $D = 2$, perturbative RG expansions [6] have not settled the issue and it seems desirable to investigate other methods.

One approach which has proven very useful in understanding phantom membranes, is the large- d limit, which can be studied exactly using standard field theoretical $1/N$ expansion techniques (here $N = d$). In this limit it was shown [7] that there is a high-temperature crumpled phase and a low-temperature flat phase which exists for $D > D_{lc} = 2 - 2/d + \dots$. In this letter we present a simple-minded attempt to extend this analysis to include some self-avoidance effects, and we will restrict ourselves to the lowest order in $1/d$ (the analysis of the fluctuations, for instance the next order in $1/d$ is in progress [8]). There are two major difficulties. The first one is that for self-avoidance to have the most interesting effect at this order, it must be long ranged, and here we will mainly consider an interaction potential $V(r(x) - r(x'))$ between monomers labelled by their internal coordinates x, x' , decaying at large distance as $V(r) \sim |r|^{-\delta}$. Such long-range repulsive forces can exist in real membranes, and in [9]

† On leave from Laboratoire de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, Paris 75231 Cedex 05, France (Laboratoire Propre du CNRS associé à l'ENS et à l'Université Paris Sud).

the effect of a Coulomb repulsion ($\delta = d - 2$) was investigated using perturbative RG methods. The second difficulty is that contrary to the usual $1/d$ expansion, mutual interactions require the introduction of a bilocal field $B(x, x')$, e.g. is a set of L^{2D} fluctuating variables where L^D is the total number of monomers, and one could worry that fluctuations around the saddle point give a L^{2D} (i.e. non-thermodynamic) contribution to the effective action at the next order in $1/d$. The approximation of considering only the lowest order in $1/d$ would amount to exchange of the order of limits $d \rightarrow \infty$ and $L \rightarrow \infty$, or equivalently allowing the distances between pairs of monomers to fluctuate (with variance $B(x, x')$) neglecting the constraints due to triangular inequalities. Preliminary results [8] suggest that this problem might be less severe than it seems†. Indeed, most of the results of this letter as a function of δ are very analogous to those of [9], which suggests that for $d > \delta$ the dependence in d is weak. However, for $D > 2$ we find new possibilities for phases and crumpling transitions not present in [9]. Finally, note that one can also interpret the present results in the spirit of a variational method consisting in finding the best Gaussian trial free energy (for the relation between the two methods, see [10]). The exponent δ then plays the role of an effective dimension. It is in this spirit that, in an early work [11], des Cloizeaux applied a very similar method to the polymer ($D = 1$) problem, which was re-examined recently [12].

We consider the following model of a tethered surface. The total energy associated to a configuration $\mathbf{r}(x)$ is the sum of the elastic energy and the potential energy due to the long-range self-avoiding force:

$$H = \int d^D \mathbf{x} \frac{\kappa}{2} (\nabla^2 \mathbf{r})^2 + \frac{\mu}{4d} (\partial_\alpha \mathbf{r} \cdot \partial_\beta \mathbf{r} - d\delta_{\alpha\beta})^2 + \frac{\lambda}{8d} (\partial_\gamma \mathbf{r} \cdot \partial_\gamma \mathbf{r} - dD)^2 + \frac{d}{2} \int d^D \mathbf{x}' V \left(\frac{(\mathbf{r}(x) - \mathbf{r}(x'))^2}{d} \right) \quad (1)$$

where κ is the bending rigidity, μ and λ are the Lamé coefficients and are kept fixed‡ when $d \rightarrow \infty$. To study the large- d limit one introduces the auxiliary fields $B(x, x')$, $s(x, x')$ and $\chi_{\alpha\beta}(x)$ in order to render the action quadratic in \mathbf{r} . The partition function can then be rewritten:

$$Z = \int D\mathbf{r}(x) \exp(-H[\mathbf{r}(x)]/T) = \int D\mathbf{r}(x) \int D\chi_{\alpha\beta}(x) DB(x, x') Ds(x, x') \exp(-H_0/T) \quad (2)$$

where the new Hamiltonian H_0 is:

$$H_0 = \int d^D \mathbf{x} \frac{\kappa}{2} (\nabla^2 \mathbf{r})^2 + \chi_{\alpha\beta} (\partial_\alpha \mathbf{r} \cdot \partial_\beta \mathbf{r} - d\delta_{\alpha\beta}) - ad\chi_{\alpha\beta}^2 - bd\chi_{\alpha\alpha}^2 + \int d^D \mathbf{x} d^D \mathbf{x}' \frac{s(x, x')}{2} \left(dB(x, x') - (\mathbf{r}(x) - \mathbf{r}(x'))^2 \right) + \frac{d}{2} V(B(x, x')) \quad (3)$$

† Most of the modes decouple at the saddle point, and one recovers a thermodynamic behaviour.

‡ Equation (1.1) differs from the usual way of writing the energy. We have performed rescalings by factors of d in order to recover the form familiar from usual $1/N$ expansions. In terms of the 'true' bending rigidity and Lamé coefficients $\kappa_0, \mu_0, \lambda_0$ one has $\kappa = \kappa_0/d, \mu = \mu_0/d, \lambda = \lambda_0/d$. The distances have also been rescaled $\mathbf{r} = d\mathbf{r}_0$, as well as the potential.

with

$$a = \frac{1}{\mu} \quad \text{and} \quad b = -\frac{\lambda}{\mu(2\mu + D\lambda)}.$$

As usual, the effective action to dominant order for $d \rightarrow \infty$ is the sum of the tree contribution (of order d) and a 1-loop contribution (also of order d) coming from integration over d degrees of freedom $\mathbf{r}(x)$. To obtain it one writes $\mathbf{r}(x) = \mathbf{r}(x)_{\text{av}} + \delta\mathbf{r}(x)$ and integrate over the fluctuating part of $\delta\mathbf{r}(x)$. One obtains:

$$H_{\text{eff}} = H_0[\mathbf{r}_{\text{av}}, \chi_{\alpha\beta}, B, s] + \frac{dT}{2} \text{Tr Ln} \left[\delta(\mathbf{x} - \mathbf{x}') \left(\frac{\kappa \nabla^4}{2} - \partial_\alpha \chi_{\alpha\beta} \partial_\beta \right) + s_c(\mathbf{x}, \mathbf{x}') \right] \quad (4)$$

where

$$s_c(\mathbf{x}, \mathbf{x}') = s(\mathbf{x}, \mathbf{x}') - \frac{1}{2} \delta(\mathbf{x} - \mathbf{x}') \int d^D y (s(\mathbf{x}, \mathbf{y}) + s(\mathbf{y}, \mathbf{x})).$$

We now look for a saddle point of the effective Hamiltonian H_{eff} . As in the analysis without self-avoidance one looks for

$$\mathbf{r}_{\text{av}}(\mathbf{x}) = d^{1/2} \zeta \mathbf{x}^\alpha \mathbf{e}_\alpha \quad \mathbf{e}_\alpha \cdot \mathbf{e}_\beta = \delta_{\alpha\beta} \quad \chi_{\alpha\beta}(\mathbf{x}) = \chi \delta_{\alpha\beta} \quad (5)$$

from isotropy and translational invariance in the internal coordinates, to which we now add

$$B(\mathbf{x}, \mathbf{x}') = B(\mathbf{x} - \mathbf{x}') \quad s(\mathbf{x}, \mathbf{x}') = s(\mathbf{x} - \mathbf{x}') \quad (6)$$

that is we assume a translationally invariant saddle point (we neglect possible boundary effects). We also assume that B and s are even functions of \mathbf{x} . One can check that this saddle point satisfies the general equations where no symmetry is assumed. Variations with respect to $B(\mathbf{x})$, $s(\mathbf{x})$, χ , ζ respectively, lead to the following saddle point equations, where $s_c(\mathbf{q}) = s(\mathbf{q}) - s(\mathbf{q} = 0)$:

$$s_c(\mathbf{q}) = \int d^D \mathbf{x} [1 - \cos(\mathbf{q}\mathbf{x})] V'(B(\mathbf{x})) \quad (7a)$$

$$B(\mathbf{x}) - \zeta^2 \mathbf{x}^2 = 2T \int \frac{d^D \mathbf{q}}{2\pi^D} (1 - \cos(\mathbf{q}\mathbf{x})) \frac{1}{\kappa q^4 + 2\chi q^2 + 2s_c(\mathbf{q})} \quad (7b)$$

$$1 - \zeta^2 + 2(a + Db)\chi = \frac{T}{D} \int \frac{d^D \mathbf{q}}{2\pi^D} \frac{q^2}{\kappa q^4 + 2\chi q^2 + 2s_c(\mathbf{q})} \quad (7c)$$

$$\chi = -\frac{1}{2D} \int d^D \mathbf{x} \mathbf{x}^2 V'(B(\mathbf{x})). \quad (7d)$$

The last equation holds *a priori* only when ζ is non-zero, i.e. in the flat phase. $B(\mathbf{x})$ is the mean-square distance per degree of freedom between two monomers of separation \mathbf{x} in the internal coordinates. A lattice cutoff $\Lambda \sim 1/a$ is implicit in all integrals.

Before analysing the phase diagram resulting from the saddle point equations 7(a-d), we recall the analysis of [7] for $V(z) = 0$. Then $s_c(\mathbf{q}) = 0$ and only equations 7(c, d) survive. In the ferromagnetic phase $\chi = 0$ and 7(c) gives

$$1 - \zeta^2 = \frac{T}{T_c} \quad \frac{1}{T_c} = \frac{1}{D\kappa} \int \frac{d^D \mathbf{q}}{2\pi^D} \frac{1}{q^2}. \quad (8)$$

Thus the membrane is flat at low temperature $T \leq T_c$. Since from (8) T_c vanishes for $D \leq 2$ the flat phase exists only in more than two dimensions. For $T \geq T_c$ the membrane is crumpled and $\zeta = 0$. The parameter $\chi > 0$ is then determined by equation 7(c). The membrane is then equivalent to a Gaussian membrane and χ is the entropic spring constant.

We now assume $V(z) > 0$, $V'(z) < 0$ for all z (monotonic self-repulsion) and $V'(z) \sim -v/z^{1+\delta/2}$ for large z . Let us first analyse the expression

$$\Gamma(\mathbf{q}) = \kappa q^4 + 2\chi q^2 + 2s_c(\mathbf{q}) \quad (10)$$

which appears in the saddle point equations and gives the effective energy of transverse fluctuations. If we assume that the radius of gyration R scale with the size L of the membrane with exponent ν , $R \sim L^\nu$, for large $|x|$, $B(x) \sim B|x|^{2\nu}$. From equation 7(a), one then finds that $s_c(\mathbf{q})$ has quite generally the following possible behaviour for small \mathbf{q} :

$$s_c(\mathbf{q}) \sim -C_0 q^{2\nu+\nu\delta-D} \quad \text{for } \nu < \frac{D+2}{\delta+2} \quad (9a)$$

$$s_c(\mathbf{q}) \sim -rq^2 + C_1 q^{2\nu+\nu\delta-D} \quad \text{for } \frac{D+2}{\delta+2} < \nu < \frac{D+4}{\delta+2} \quad (9b)$$

$$s_c(\mathbf{q}) \sim -rq^2 - \frac{q^4}{4!D(D+2)} \int d^D x x^4 V'(B(x)) + \dots \quad \text{for } \frac{D+4}{\delta+2} < \nu \quad (9c)$$

where C_0, C_1 , are positive constants. We have defined

$$r = -\frac{1}{2D} \int d^D x x^2 V'(B(x)).$$

C_1 is equal to $vB^{-(1+\delta/2)}A_\nu$, where

$$A_\nu = \int d^D y \frac{(y^2/2 - 1 + \cos(y))}{y^{\nu(2+\delta)}}.$$

Note that the self-energy $s_c(\mathbf{q})$ of the fluctuations associated with the repulsive potential is *negative* for all \mathbf{q} and by itself would lead to an instability (this is the case for situation (9a) which will not occur and we need only consider (9b) and (9c)). The q^2 term has a negative coefficient $-r$ because repulsion between the monomers pushes each of them away from the plane of the membrane. This does not mean, however, that the membrane is unstable, because the elastic term χq^2 compensates for it. In the flat phase $\chi = r$, and more generally $\chi \geq r$. One easily sees that $\delta s_c(\mathbf{q}) = s_c(\mathbf{q}) - rq^2$ is always *positive*, guaranteeing the stability. Similarly the q^4 term in the expansion (9c) is *positive* because self-avoidance is unfavourable to bent configurations.

We now list the possible phases when D and δ are varied. Their domain of existence is represented in figure 1. We restrict ourselves to $\delta > D$ (the case $\delta < D$ leads to a super-stretched flat phase or to an instability as discussed in [9]). Let us first consider the case $\zeta = 0$. Then there are three possible isotropic phases:

Crumpled 1, for which $\Gamma(\mathbf{q}) \sim q^2$, e.g. $\chi > r$. This implies from (7b) that $\nu = (2-D)/2$, which itself is possible from (9) only above a critical 'dimension' when $\delta > \delta_c(D)$ where $\delta_c(D) = 4D/(2-D)$ corresponds to twice the fractal dimension of a single crumpled membrane (above which intersections are irrelevant). This regime corresponds to the ordinary crumpled phase where self-avoidance is irrelevant. For $D > 2$ self-avoidance is always relevant and this regime disappears.

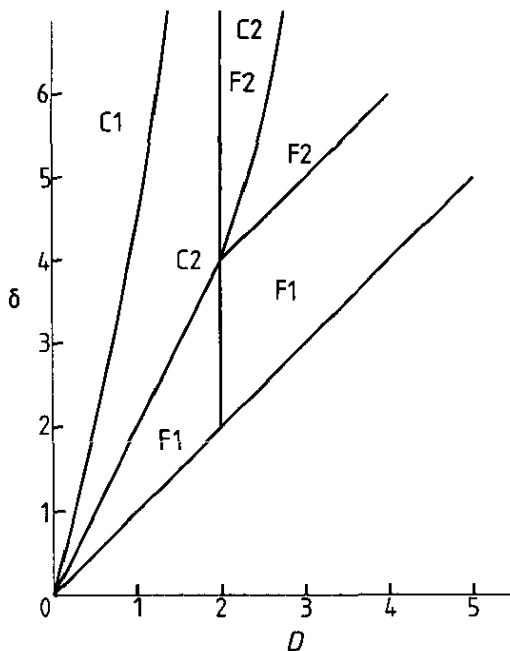


Figure 1. Possible phases in the plane (D, δ) in the limit $d \rightarrow \infty$. C1 denotes the crumpled phase where self-avoidance is irrelevant. C2 is a swollen crumpled phase with $\nu = 2D/\delta$. In addition, C3 with $\nu = (4 - D)/2$ corresponds to the crumpling transition and exists only in the region where F2 and C2 coexist. F1 is a flat phase with anomalous roughening and F2 a flat phase with normal roughening.

Crumpled 2, for which $\Gamma(q) \sim q^{2\nu+\nu\delta-D}$, i.e. $\chi = r$. This implies from (7b) that $\nu = 2D/\delta$. This regime corresponds to a crumpled phase where self-avoidance is relevant with a non-trivial exponent. It generalizes the result of des Cloizeaux [11] for the polymer. This regime corresponds to case (9b) above and thus is possible only when $\delta_{c1}(D) < \delta < \delta_c(D)$ where $\delta_{c1}(D) = 4D/(4 - D)$. Note that this is twice the fractal dimension of the network at the crumpling transition to this order. Furthermore this regime is possible only for $\nu < 1$ or equivalently $\delta > 2D$, since otherwise the momentum integral in (7c) diverges. Thus, as represented on figure 1, for $D < 2$ this phase is bounded by the line $\delta = 2D$, below which the only solution is a flat phase (see below), and for $D > 2$ by the line $\delta = \delta_{c1}(D)$. The amplitude is

$$B = \left(\frac{\nu}{T} \frac{A_\nu}{A'_\nu} \right)^{2/\delta}$$

where

$$A'_\nu = \int d^D y \frac{(1 - \cos(y))}{y^{\nu(2+\delta)-D}}$$

Crumpled 3, for which $\Gamma(q) \sim q^4$ (also $\chi = r$). This implies from (2.8) that $\nu = (4 - D)/2$. This regime also corresponds to a crumpled phase where self-avoidance is relevant, but with a different exponent than the other one. In this phase the bending energy

dominates the fluctuations and the effective bending rigidity is

$$\kappa + \int d^D x \frac{x^4}{4! D(D+2)} V'(B(x)).$$

As a consequence the exponent ν corresponds to the thermal roughening exponent of a flat membrane. This phase does not appear in the analysis of [9]. It corresponds to the membrane being at the crumpling transition $T = T_c$, which is also an isotropic state. This regime corresponds to case (9c) above and thus is possible only when $\delta > \delta_{c1}(D) = 4D/(4-D)$. Again this regime is possible only for $\nu < 1$ which is equivalent to $D > 2$.

Let us now consider the flat phases. In the flat phase $B(x) = \zeta^2 x^2 + B' x^{2\nu'}$, where $\nu' < 1$ is the exponent of transverse roughening (usually called ζ [3]). From (7d) the q^2 terms exactly cancel and $\chi = r$. There are two possible cases:

Flat 1, for which $\Gamma(q) \sim q^{2+\delta-D}$. From (9), since $\nu = 1$, this implies that this phase exists only for $D < \delta < D+2$. It has an anomalous transverse roughening exponent $\nu' = (2 + \delta - 2D)/2$. The range of variation of ν' is thus between $(2 - D)/2$ and $(4 - D)/2$. Furthermore the self-consistency that the RHS of (7b) is subdominant gives $\delta < 2D$ (or equivalently the condition that the integral in (7c) is finite). Thus this phase can survive for $D < 2$.

Flat 2, for which $\Gamma(q) \sim q^4$. This is possible for $\delta > D+2$. In this phase, the roughening is normal with $\nu' = (4 - D)/2$ and self-avoidance is irrelevant. The lower critical dimension is $D = 2$.

We have listed the possible phases. A complete study of equation (7) and of the transitions will be presented in [8]. A simple argument for the existence of a crumpling transition in the region $D > 2$, $\delta > 4D/(4-D)$ is as follows. A ferromagnetic phase must exist for small temperatures since from (7) one has:

$$1 - \zeta(T)^2 \sim T \int \frac{d^D q}{2\pi} \frac{1}{\kappa q^2 + \delta s_c(q)}. \quad (10)$$

We have supposed infinite elastic constants for simplicity and thus $\zeta(T=0) = 1$. In the above, $\delta s_c(q) = \int d^D x (1 - \cos(x \cdot q) - (x \cdot q)^2/2) V'(B(x))$. Thus for T small it is clear that $\zeta > 0$ if there is no infrared divergence in the RHS, i.e. if $D > \min(\delta - D, 2)$. Similarly, if we suppose $\kappa = 0$ for simplicity, the crumpled phase 2 must exist at high T . The RHS of (7c) must be large enough so that (7c) can be satisfied for $\zeta = 0$. Since the RHS represents the RMS of the angle fluctuations this can be likened to a Lindemann criterion for the normals to the surface. The idea is that χ can be considered as setting the scale and for large T can be made $\sim T$ to satisfy (7c). More precisely, following a similar analysis by des Cloizeaux [11], one can rescale out all dimensional quantities, and find that in the crumpled 2 phase:

$$\Gamma(q) = 2\chi q^2 A(qw^{-\beta})$$

where

$$w \sim \frac{\nu}{\chi} \left(\frac{T}{\chi} \right)^{-(1+\delta/2)} \quad \beta = 2/(4D + (D-2)\delta) \quad A(\infty) = 1$$

and $A(z)$, as in [11], satisfies rescaled integral equations analogous to (7) (for small z , $A(z) \sim z^{D-2+4D/\delta}$). This scaling form describes the crossover in momentum from a free membrane regime for $q \gg w^\beta$ to a long-distance regime where self-avoidance is relevant $q \ll w^\beta$. Of course one must have $w^{-\beta} \gg a$ for the continuum description (7) to be correct, which we assume here. If this is not the case, more microscopic details are necessary, and in particular one should use the discretized form $1 - \cos(ka)$ for the free propagator [8]. Using the above form one finds that (7c) becomes:

$$1 = \left(\frac{\nu}{T}\right)^{2/\delta} \int_{q \ll w^\beta} d^D q \frac{q^2}{q^{D+4D/\delta}} + \frac{T}{\chi} \int_{q \gg w^\beta}^\Lambda d^D q.$$

A solution $\chi \sim T$ exists to this equation (such that the first term is small for large T).

An interesting feature of $D \geq 2$ is that, contrarily to the polymer case, short-range repulsive potentials $V(z)$ do have an effect even in the limit $d \rightarrow \infty$. This is because for the phantom membrane $R \sim a^{(2-D)/2}$ for $D > 2$ which from (7a) cannot be preserved even by short-range repulsion (except maybe if $V(z) = 0$ for $z > Ca$). The possible phases for short-range potentials, such as for instance exponentially decaying potentials are the following. Crumpled 3 is possible with $\nu = (4 - D)/2$: one is then in case (9c) with $\Gamma(q) \sim q^4$ since clearly the first two moments exist. Note that the equation determining the amplitude B involves short-distance details. Crumpled 2 should be possible by letting $\delta \rightarrow \infty$ although involves logarithms. Finally the normal flat 2 phase is also possible.

Similarly a long-range attractive potential leads, for $\delta > 4D/(2 - D)$, to two possible phases, one where $\nu = (2 - D)/2$ corresponding to regime (9b) and the other, contracted, with $\nu = 2D/\delta$, corresponding to (9a) with a negative C_0 . For $\delta < 4D/(2 - D)$ there is no obvious solution, which indicates that there might be a collapsed phase.

To conclude, we have investigated the lowest order of a $1/d$ approach to tethered surfaces including self-avoidance. For long-range repulsion we found many possible regimes depending on δ and D . Although the physics of long-range interactions is different this might provide some insight into the short-range problem as well. Note that for $D = 2$, $\delta = 3$ we find $\nu' = \frac{1}{2}$ (numerical simulations [3] with short-range self-avoidance give $\nu' = 0.65$). We found that short-range repulsion also leads to non-trivial solutions. Fluctuation calculations leading to higher-order $1/d$ corrections [8] might thus give interesting information. One can anticipate that the flat phases be stabilized, thus moving the line $D = D_{lc} = 2$ figure 1 to the left.

I thank D R Nelson for discussions. This work was supported by NSF grant DMS-9100383.

Note added. The present results, interpreted as in a variational method for short-range self-avoidance (δ being the dimension), are in surprisingly good agreement with recent numerical simulations of $D = 2$ membranes in high dimensions by Grest [13]. The agreement is better than for the polymer $D = 1$ [11], and much better than for the Flory value for membranes [2]. The measured value ν_m from simulations, the value ν_p predicted here and the Flory value ν_F are:

for $\delta = 3$: $\nu_m = 1$, $\nu_p = 1$, $\nu_F = 0.80$,
 for $\delta = 4$: $\nu_m = 1$, $\nu_p = 1$, $\nu_F = 0.66$,
 for $\delta = 5$: $\nu_m = 0.82$, $\nu_p = 0.80$, $\nu_F = 0.57$,
 for $\delta = 6$: $\nu_m = 0.69$, $\nu_p = 0.66$, $\nu_F = 0.50$,
 for $\delta = 8$: $\nu_m = 0.6$, $\nu_p = 0.50$, $\nu_F = 0.40$.

References

- [1] Kantor Y, Kardar M and Nelson D R 1986 *Phys. Rev. Lett.* **57** 791
Kantor Y and Nelson D R 1987 *Phys. Rev. Lett.* **58** 2774
- [2] Paczuski M, Kardar M and Nelson D R 1988 *Phys. Rev. Lett.* **60** 2638
- [3] Abraham F F, Rudge W E and Plischke M 1989 *Phys. Rev. Lett.* **62** 1757
Abraham F F and Nelson D R 1990 *J. Physique* **51** 2653
- [4] Wen X *et al* 1991 *Preprint* MIT
Schmidt C 1991 private communication
- [5] Baumgartner A and Renz W 1991 *Europhys. Lett.* (submitted)
Baumgartner A 1991 *J. Physique* **1** 1 1549
- [6] Kardar M and Nelson D R 1987 *Phys. Rev. Lett.* **58** 1289
- [7] David F and Gutter E 1988 *Europhys. Lett.* **5** 709
Gutter E, David F, Leibler S and Peliti L 1989 *J. Physique* **50** 1789
- [8] Le Doussal P 1992 (in preparation)
- [9] Kantor Y and Kardar M 1989 *Europhys. Lett.* **9** 53
- [10] Mézard M and Parisi G *J. Phys. A: Math. Gen.* **19** 444
- [11] des Cloiseaux J 1970 *J. Physique* **31** 715
- [12] Bouchaud J P, Mézard M, Parisi G and Yedidia J 1991 *Preprint* LPS-ENS, Paris
- [13] Grest G S 1991 *J. Physique* **1** 1 1695